## Impulse response

So far circuits have been driven by a DC source, an AC source and an exponential source. If we can find the current of a circuit generated by a Dirac delta function or impulse voltage source $\delta$, then the convolution integral can be used to find the current to any given voltage source!

## Example Impulse Response

The current is found by taking the derivative of the current found due to a DC voltage source! Say the goal is to find the $\delta$ current of a series LR circuit ... so that in the future the convolution integral can be used to find the current given any arbitrary source.

Choose a DC source of 1 volt (the real Vs then can scale off this). The particular homogeneous solution (steady state) is 0 . The homogeneous solution to the non-homogeneous equation has the form:

Assume the current initially in the inductor is zero. The initial voltage is going to be 1 and is going to be across the inductor (since no current is flowing):
$v(t)=L \frac{d i(t)}{d t}: v(0)=1=L *\left(-\frac{A R}{L}\right):$
If the current in the inductor is initially zero, then:
Which implies that:
So the response to a DC voltage source turning on at $\mathrm{t}=0$ to one volt (called the unit response $\mu$ ) is:

$$
i_{\mu}(t)=\frac{1}{R}\left(1-e^{-\frac{t}{\frac{L}{R}}}\right)
$$

Taking the derivative of this, get the impulse ( $\delta$ ) current is:

Now the current due to any arbitrary $\mathrm{V}_{\mathrm{S}}(\mathrm{t})$ can be found using the convolution integral:

Don't think $\mathrm{i}_{\delta}$ as current. It is really $\frac{d}{d t} \frac{\text { current }}{1 \text { volt }} . \mathrm{V}_{\mathrm{S}}(\tau)$ turns into a multiplier.

## LRC Example

Find the time domain expression for $\mathrm{i}_{0}$ given that $\mathrm{I}_{\mathrm{s}}=\cos (\mathrm{t}+\pi / 2) \mu(\mathrm{t}) \mathrm{amp}$.

Earlier the step response for this problem was found:

The impulse response is going to be the derivative of this:
$i_{o}(t)=\int_{0}^{t} i_{o_{\delta}}(t-\tau) I_{s}(\tau) d \tau+C_{1}$

The Mupad code to solve the integral (substituting $x$ for $\tau$ ) is:

```
f := exp(-(t-x)) *sin(t-x) *(1 + cos(x));<br>S := int(f,x = 0..t)
```


## Finding the integration constant

This implies:

