

Impulse Response

So far circuits have been driven by a DC source, an AC source and an exponential source. If we can find the current of a circuit generated by a Dirac delta function or impulse voltage source δ , then the convolution integral can be used to find the current to any given voltage source!

Example Impulse Response

The current is found by taking the derivative of the current found due to a DC voltage source! Say the goal is to find the δ current of a series LR circuit ... so that in the future the convolution integral can be used to find the current given any arbitrary source.

Choose a DC source of 1 volt (the real V_s then can scale off this). The particular homogeneous solution (steady state) is 0. The homogeneous solution to the non-homogeneous equation has the form:

$$i(t) = Ae^{-\frac{t}{R}} + C$$

Assume the current initially in the inductor is zero. The initial voltage is going to be 1 and is going to be across the inductor (since no current is flowing):

$$v(t) = L \frac{di(t)}{dt}$$

$$:v(0) = 1 = L * \left(-\frac{1A}{L}\right)$$

$$:A = -1/R$$

If the current in the inductor is initially zero, then:

$$i(0) = 0 = A + C$$

Which implies that:

$$C = -A = 1/R$$

So the response to a DC voltage source turning on at $t=0$ to one volt (called the unit response μ) is:

$$i_{\mu}(t) = \frac{1}{R}(1 - e^{-\frac{t}{R}})$$

Taking the derivative of this, get the impulse (δ) current is:

$$i_{\delta}(t) = \frac{e^{-\frac{t}{R}}}{L}$$

Now the current due to any arbitrary $V_s(t)$ can be found using the convolution integral:

$$i(t) = \int_0^t i_{\delta}(t-\tau)V_s(\tau)d\tau = \int_0^t f(t-\tau)g(\tau)d\tau + C_1$$

Don't think i_{δ} as current. It is really $\frac{\text{current}}{dt \cdot 1\text{volt}}$. $V_s(\tau)$ turns into a multiplier.

LRC Example

Find the time domain expression for i_o given that $I_s = \cos(t + \pi/2)\mu(t)$ amp.

Earlier the step response for this problem was found:

$$i_{o\mu} = \frac{1}{2}(1 - e^{-t}(\cos t + \sin t))$$

The impulse response is going to be the derivative of this:

$$i_{os} = \frac{di_{o\mu}}{dt} = 0 + \frac{1}{2}e^{-t}(\cos t + \sin t) - \frac{1}{2}e^{-t}(-\sin t + \cos t)$$

$$i_{os} = \frac{1}{2}e^{-t}(\cos t + \sin t + \sin t - \cos t) = e^{-t} \sin t$$

$$I_s = 1 + \cos t$$

$$i_o(t) = \int_0^t i_{os}(t - \tau) I_s(\tau) d\tau + C_1$$

$$i_o(t) = \int_0^t e^{-\nu(t-\tau)} \sin(t - \tau)(1 + \cos \tau) d\tau + C_1$$

$$i_o(t) = \frac{\cos t}{5} + \frac{2 \sin t}{5} - \frac{7e^{-t} \cos t}{10} - \frac{11e^{-t} \sin t}{10} + \frac{1}{2} + C_1$$

The Mupad code to solve the integral (substituting x for τ) is:

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f := exp(-(t-x)) *sin(t-x) *(1 + cos(x));<br>S := int(f,x = 0..t)
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Finding the integration constant

$$i_o(0_+) = 0 = \frac{1}{5} - \frac{7}{10} + \frac{1}{2} + C_1$$

This implies:

$$C_1 = 0$$